RELAXATION-TYPE SELF-OSCILLATIONS ARISING DURING BURNING OF POWDER IN A HALF-CLOSED VOLUME

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The possible occurrence of oscillatory phenomena during the burning of powder has been indicated in [1-4]. The problem of self-oscillations under constant pressure and of forced oscillations under variable pressure has been examined in a close-to-harmonic approximation in [2-3]. The results of these papers, however, are applicable only to such burning conditions for which the corresponding problem is weakly nonlinear. For strong nonlinear relationships, i.e., conditions particularly characteristic for nonstationary burning processes, a solution to the analogous problem was not obtained. In the following, an attempt to fill this gap to some degree is made for the case in which the powder burns in a half-closed volume, while the oscillations are close to being discontinuous.

In dimensionless form, the equations describing the burning of powder in a half-closed volume, assuming a linear relation between the stationary burning rate and the initial temperature, have the form

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial\xi}\left(\frac{\partial\theta}{\partial\xi}+\omega\theta\right),\qquad \gamma\frac{d\pi}{d\tau}=\omega-\mu\pi,$$

$$\omega=\pi^{\nu}\frac{1+\beta}{2}\left[1+\left(1-\frac{4\beta}{\pi^{\nu}(1+\beta)^{2}}\left|\frac{\partial\theta}{\partial\xi}\right|_{\xi=0}\right)^{1/2}\right],$$

for the following initial and boundary conditions:

$$\pi=1,\quad \omega=1,\quad \theta=e^{-\xi}\quad \text{for}\quad \tau=0,$$

$$\theta(0,\tau)=1,\qquad \theta(\infty,\tau)=0,\qquad \left.\frac{\partial\theta}{\partial\xi}\right|_{\xi=\infty}=0.$$

Here, θ, π, ω, ξ, and τ all dimensionless, are the temperature, pressure, burning rate, spatial coordinate, and time, respectively; β is a parameter which characterizes the level to which the powder is heated; γ is the ratio of the gas evolution time to the solid state relaxation time under initial pressure; μ is the ratio of the mass velocity of gas evolution for a given nozzle cross section and initial pressure to the gas mass evolved per unit time by the entire powder area under steady-state conditions at the same pressure; and ν is the exponent in the expression which describes the steady-state burning rate as a function of the initial temperature and pressure.

We use the method of integral relations [6-9] to solve the heat-conduction equation. The function $\theta(\xi,\tau)$ is sought in the form

$$\theta(\xi,\tau)=[1-f(\tau)]e^{-\xi}+f(\tau)e^{-\omega(\tau)\xi},$$

where $f(\tau)$ is a certain bounded function. The use of this form is clearly advantageous, for example, in the case of relatively slow varying velocity and pressure. As a result of computations typical for the method of integral relations [6], the initial problem reduces to the solution of a system of equations of the form

$$\gamma\frac{dy}{d\tau}=\frac{B}{A}-y,\qquad \frac{d\pi}{d\tau}=y,$$

$$A=(\gamma y+\pi)^{2}-\pi^{\nu}\beta,\qquad B=(\gamma y+\pi)^{3}(\nu\pi^{-1}y-\gamma y+\pi^{\nu}-\pi).\quad (1)$$

System (1) has two steady states

$$y_1^\circ=0,\quad \pi_1^\circ=0;\quad y_2^\circ=0,\quad \pi_2^\circ=1.$$

Let us assume that $\gamma \ll 1$. Then we obtain two differential equations with a small parameter in front of the derivative. It is well known [10] that relaxation oscillations can arise in such a system. The behavior (characteristic for relaxation processes) of the phase point in the phase space can be represented in the following way. If the phase point is situated at a point remote from the steady-state curve of the first equation in (1)

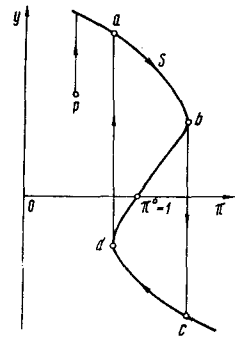

Fig. 1

$$B/A-y=0,\qquad (2)$$

then the variable y varies rapidly, since

$$\frac{dy}{d\tau}=\frac{1}{\gamma}\left(\frac{B}{A}-y\right).$$

At the same time, the variable π remains constant in the first approximation. Indeed, by performing the substitution $t=\tau/\gamma$, instead of (1), we get

$$\frac{dy}{dt}=\frac{B}{A}-y,\qquad \frac{d\pi}{dt}=\gamma y.\qquad (3)$$

In other words, the derivative $d\pi/dt$ is roughly zero, whereas the derivative dy/dt differs appreciably from zero (the phase point is situated at a point remote from the curve S). The variation of y and π will be of the same nature until the function $A/B-y$ approaches zero, i.e., until the phase point comes to lie in the γ-vicinity of one of the stable steady states of the equation of rapid motion. As soon as this takes place, the variables y and π begin to vary at comparable rates. As a result, the steady state defined by Eq. (2) will shift, and the phase point of system (1) will, in its motion, accompany this shift of the steady state in the phase space. The nature of the motion varies abruptly as soon as the separation point has been reached [11], at which

$$\partial(B/A)/\partial y-1=0.$$

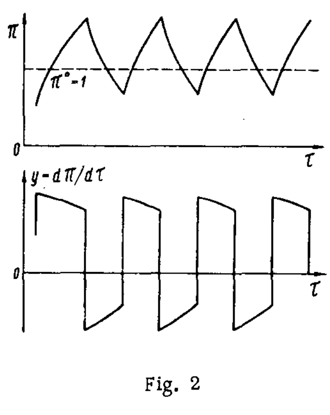

Fig. 2

The variable y again will vary rapidly until the phase point approaches the other stable steady state of Eq. (2). One of the possible variants of phase representation in shown in Fig. 1, where P is the initial position of the phase point, and b and d are separation points. Figure 2 shows the plots of π and y vs. time that correspond to this case.

In order that a limiting cycle (the contour a, b, c, d) exist in our system, it is sufficient to satisfy the following conditions:

1) When curve S passes across the steady state of system (1), y must change its sign.

2) The steady state must be unstable.

3) The straight line $\pi = \pi^\circ$ must:

a) either cut curve S not only at the singular point, but at least at two more points at which y is of different sign, in which case the derivatives $\partial (B/A)/\partial\pi$ at these points must be strictly less than zero;

b) or must cut curve S at least at one point (where $\partial (B/A)/\partial\pi < 0$), while the steady state must be doubly degenerate.

Condition b) is limiting for case a) when the singular point is a cuspidal point of the first kind.

Since the steady state $\pi^\circ = 0$ corresponds to the absence of burning in the chamber, the following analysis will be performed in the proximity of the singular point $\pi^\circ = 1$.

It is not difficult to obtain relationships between the parameters β and ν, which, if satisfied, lead to wave (periodic) solutions. Let us assume that for $\tau = 0$, the system is in an unstable steady state $\pi^\circ = 1$. Neglecting the terms with γ^4 and γ^3, from Eq. (2), we find

$$y^3\gamma^2 (1 - 3\nu) + y^2\gamma (2\pi + 3\nu\pi + 6\gamma\pi^2 - 3\gamma\pi^{\nu+1}) + \\ + y (\pi^2 - \pi^\nu\beta + 4\gamma \pi^3 - 3\gamma\pi^{\nu+2} - \nu\pi^2) + \pi^4 - \pi^{\nu+3} = 0. \quad (4)$$

Conditions 1)-3) are satisfied if, after substitution of $\pi = 1$ into Eq. (4), this equation will either have two real roots of different sign, or a double zero root. In the first case, $\nu + \beta > 1$ and $\nu < 1/3$ must be fulfilled (with an accuracy to terms containing γ), and in the second case, $-\beta + \nu = 1$ must be fulfilled for any ν. The maximum and minimum values of the oscillation amplitude can be determined either from the condition of a zero determinant of Eq. (4), or from the condition [11]

$$\frac{\partial B / A}{\partial y} = 1, \qquad \frac{B}{A} - y = 0.$$

Neglecting the time of rapid motions, the oscillation period T is determined from the relation [10]

$$T = \int_{y_{\min}}^{y_{\max}} \frac{d\pi}{y} + \int_{-y_{\min}}^{-y_{\max}} \frac{d\pi}{y},$$

where y is a root of Eq. (4), while $\pm y_{\max}$ and $\pm y_{\min}$ are obtained from the same equation by substituting $\pi = \pi_{\max}$ and $\pi = \pi_{\min}$.

The results obtained should be treated as qualitative for the following reasons:

1) as a functional relation between ω, π, and $\partial\theta/\partial\xi$ we have taken an expression that corresponds to a linear dependence of the steady-state burning rate on the initial temperature, which is not always the case;

2) the initial system of equations was solved under the conventional assumptions regarding the burning process at the powder surface at a constant temperature, which in itself is a rough approximation; and

3) the method of integral relations, employed in the analysis, is an approximate method, whose accuracy cannot be assessed without a computer.

It should be noted that assumptions 1) and 2) can be removed at the expense of complicating the computations. In principle, assumption 3) can be also eliminated, since in each concrete case, the approximate solution may be made to approach closely the exact solution by synthesizing the appropriate relation for θ. As a final comment we wish to warn against making the error of assuming that, owing to its smallness, the parameter γ has an insignificant influence on the characteristic of the process. The point is that the critical value γ_* [1] may in its turn be very small and, consequently, regardless of the condition $\gamma \ll 1$, the parameter γ will continue to be a factor decisively affecting the steadiness or unsteadiness of burning.

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